Are Defined Contribution Pension Plans Fully Funded?
Aligning Incentives Through a Hedgeable Liability Measure

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Abstract

Most pension plans either have Defined Benefits (DB) with unhedgeable liabilities, which almost inevitably create deficits, or have Defined Contributions (DC) but no liabilities or commitments in terms of retirement income for their affiliates. This lack of an explicit retirement income objective in DC arrangements removes the incentives for pension fund managers to control the primary investment risk faced by beneficiaries: the variations in the “exchange rate” of accumulated assets to retirement income. To align the incentives of pension fund managers with the replacement income objective of their beneficiaries, we propose a hedgeable definition of pension liability for DC plans and derive related performance metrics, manager payment functions, and asset allocation strategies that foster retirement security. The proposed strategies also generate relevant signals for pension fund beneficiaries’ saving/consumption decisions during the accumulation phase.

Keywords: target date funds, target income, pension funds, manager incentives, asset allocation.

1 Introduction

A massive transition from defined-benefit (DB) to defined-contribution (DC) pension schemes has been taking place globally in the last three decades, both in nationwide public pension schemes as well as corporate pension plans. The trend from DB to DC pension plans transfers the responsibility for retirement security to individuals, and as a consequence a large portion of the population is unlikely to be able to sustain their standard of living in retirement (Poterba, 2014). That transition has been fueled by the widespread defunding events observed in public and corporate DB plans in the United States (Novy-Marx and Rauh, 2011; Merton, 2014) and elsewhere (OECD, 2016; Mitchell and Piggott, 2016). This situation has occurred in spite of the development of sound liability-driven techniques to manage portfolios (e.g. Sharpe and Tint, 1990; Rudolf and Ziemba, 2004; Detemple and Rindisbacher, 2008; Martellini and Milhau, 2012, and references therein) and despite the fact that the sponsoring institution in DB plans has a

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1Recent data show that in the United States, more than 70% of pension schemes follow a DC design, including employer-sponsored and individual saving accounts (Investment Company Institute, 2021). Globally, assets in DC plans surpassed DB assets for the first time in the seven largest pension markets in 2018 (Thinking Ahead Institute, 2019).
strong incentive to manage assets in a way that avoids shortfalls relative to the promised benefits, because the latter constitute a liability for the institution. We argue that the systematic failure to avoid underfunding in DB plans is caused by a structural problem in the current definition of pension liabilities, which can be surmounted. Indeed, the way retirement benefits are usually defined in DB plans creates unhedgeable liabilities because the level of accrued benefits has a loose relation with the contributed funds.\footnote{In a typical DB plan for U.S. state employees, “an active worker accrues the right to a periodic benefit upon retirement that equals a flat percentage of his final (or late-career) salary times his years of service with the employer” (Novy-Marx and Rauh, 2011). Hence, the pension benefit depends on contribution time, and the latter only has a loose relationship with the level of contribution dollars needed to finance the promised benefits.}

On the other hand, there is accumulating evidence that pension and investment risks are not properly managed within DC funds, as pointed out by several works such as Impavido et al. (2010), Merton (2014), Nijman and van Soest (2019), and van Bilsen et al. (2020). For example, the strategies followed by the widespread target date funds, use as “safe” assets (relatively) short-term government bonds, which reflect an asset-only perspective in their design, instead of focusing on retirement benefits.\footnote{For instance, the bond benchmark index used by target date funds, such as the LifePath Index 2055 Fund of BlackRock, which is the Bloomberg Barclays US Aggregate Bond Index, presents a duration fluctuating around an average of 4.9 years between 1978 to 2019. Blackrock reports $200 billion in “LifePath Target Date fund’s assets.”} While investment in short-term bonds commands low volatility in terms of asset value, the level of pension benefits that such investment can yield is as volatile as an investment in an equity index (see Merton, 2014, and Section 4.1).\footnote{To illustrate this point, Merton (2014) presents an example of a worker with $1 million in retirement assets invested in T-bills. If these assets are converted into an inflation-protected annuity with a return of 4% to 5% he will receive “an annual income of $40,000 to $50,000. But now suppose he retires a few years later, when the return on the annuity has dropped to 0.5%. His annual income will now be only $5,000” plus inflation adjustments during the rest of his life.} In other words, investment strategies in DC funds with an asset-only focus leave affiliates exposed to the main investment risk in the retirement problem (i.e., interest rate risk). Consequently, pension benefits remain very uncertain even towards the end of the accumulation period. This mismatch between the duration of DC funds’ bond portfolios and the terms of the pension cashflows is referred to as the “duration puzzle in life-cycle investment” (van Bilsen et al., 2020).

It is highly plausible that the explanation for the existence of the duration puzzle is the inadequacy of current incentives in DC systems. Indeed, regulatory incentives for pension fund managers of DC plans, such as performance metrics and payment functions, usually have an asset-only focus. For example, the management fees of managers in DC plans in most cases are either a fixed proportion of assets under management (AUM) or a fixed portion of mandatory contributions, usually proportional to labor income (see Han and Staňko, 2020). In some cases,
management fees are complemented with a “performance fee” proportional to asset returns in excess of a benchmark portfolio, whenever the excess return is positive.

The missing focus on retirement income in DC plans contrasts with the liability-driven strategies and performance metrics fund managers use in DB systems. In the absence of a formal definition for pension liabilities in DC plans (which are no more than the explicit expression of a target level of income benefits for the affiliates) and the current asset-based payment functions of DC pension fund managers, it is perhaps not surprising that investment risks are not necessarily managed to the best interests of the beneficiaries. After all, the emergence of sound liability-driven investment and risk management strategies in the DB pension space are intimately linked to the presence of explicit pension liabilities, as it leads to the use of the assets-to-liability ratio, or the surplus (i.e., assets minus liabilities), as performance metric.

In this context, we address central problems in DB and DC systems by introducing a new pension liability definition that is hedgeable, and fosters retirement security by aligning the incentives of pension fund managers, with the replacement income goals of beneficiaries in several ways. First, the liability naturally allows a correct measurement of investment performance and risk in DC plans through a funding ratio (or funded status) and its variations (see Section 2). Furthermore, in Section 3.1, we propose liability-driven payment functions for pension fund management companies, that leads to the key incentive alignment, and motivated by important conclusions from contract theory. In Section 3.2 we introduce a new class of liability-driven asset allocation rules called target income strategies, which secure a strictly increasing level of retirement income benefits, known during the accumulation phase, within DC plans. Section 4.2 presents an empirical analysis comparing the target income strategies with the typical glide-path allocation of target date funds. We find that the benefits in terms of risk management (e.g., derisking as retirement approaches and securing gains in retirement benefits during the accumulation phase) are substantial and consistent. In Section 5 we discuss the proposed target income strategies’ characteristics, in light of the lessons from the optimal life-cycle portfolio selection literature. Section 6 concludes, and the appendix collects technical derivations.

For instance, according to their investment policy statement, the U.S. Pension Benefit Guaranty Corporation “Utilize Liability Driven Investment (LDI) techniques to minimize funded status volatility and the risk of future deficits” (PBGC, 2019).
A Hedgeable Liability Definition for DC Plans

The first step towards effective risk management in pension funds is to define a safe asset for the accumulation period with respect to the retirement benefits that it generates. In accumulation, the proper safe asset is well-known to be an inflation-linked deferred annuity, designed to deliver a lifetime replacement income stream in retirement that guarantees a fixed purchasing power in terms of consumption goods (see Yaari, 1965; Brown, 2001). While they can be regarded as safe assets for retirement, the observed demand for annuities remains low. To try and understand this so-called “annuity puzzle,” Brown (2001) recognizes that a life-cycle portfolio choice model only partially explains the empirical choices of individuals concerning annuitization (Brown, 2001). In a related effort, Pashchenko (2013) surveys various reasons that explain the low level of annuitization, such as the existence of “pre-annuitized” wealth (Social Security and DB plan benefits), adverse selection ruling out groups with higher mortality, minimum investment requirements, high surrender charges, and the perceived cost-inefficiency of annuities, which are not sold at an actuarially fair price (Friedman and Warshawsky, 1988, 1990), the fact that they do not contribute to bequest objectives, and the fact that annuitized wealth cannot be recovered in the form of capital even if the beneficiary experiences a severe health problem that would generate large expenses (Peijnenburg et al., 2017). As a result of these limitations, if deferred inflation-linked annuities are, in principle, the safe retirement assets, they cannot be regarded as such in practice.

Against this backdrop, one may use so-called retirement bonds as a proxy for the safe retirement asset (Martellini and Milhau, 2020). A retirement bond is a fixed-income security that pays $1 of replacement income every period, e.g., month or year, subject to a cost of living adjustment (COLA), starting at the retirement date $T$, for a period given by the life expectancy of the cohort after retirement, e.g., $M = 20$ years, thus providing replacement income, say from age 65 to age 85 (see Merton and Muralidhar, 2017, for the case of a closely related type of bond called retirement SeLFIES).

One key difference between retirement bonds and deferred annuities is that the latter offer longevity risk protection while the former does not. This is not a severe limitation, however, since protection against extreme longevity risk beyond the maturity date of the retirement bond for a given retiree reaching age 85 can be achieved by purchasing a deferred late-life annuity at

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6The life expectancy at age 65 for a generic US individual was 19.5 years in 2018 (see Table B in National Vital Statistics Reports, 2021: Arias et al., 2021), hence 20 years after rounding.
retirement date in individual DC plans (Horneff et al., 2020), or by risk-sharing within members in collective decumulation solutions (Milevsky and Salisbury, 2015; Fullmer, 2019). In principle, if enough people of a given cohort held retirement bonds, as a group, they could exchange such bonds for life (indexed) annuities with an insurance company with a rate close to one-to-one in terms of real dollars of retirement. This is because such bonds would represent a hedging asset for an insurance company issuing such annuities for individuals in that cohort because the insurer can use the cash-flows of the bonds it obtains from the individuals that die before the life expectancy of the cohort to pay the annuity cash-flows of the individuals that survive past the life expectancy. In this sense, the price of the retirement bond is a proxy for the cost of financing one dollar of consumption throughout the life of each individual after retirement, taking into account the potential risk-pooling benefits that can be tapped into the payout phase.

Retirement bonds are currently not traded in fixed-income markets. However, their fair price at any date \( t \) during the accumulation phase \( (B_t) \), can be obtained by standard no-arbitrage arguments; that is, by discounting future replacement income cash flows at the zero-coupon rates of appropriate maturities (See Appendix A for details about the pricing of retirement bonds). That retirement bonds can be priced by arbitrage arguments implies that their payoffs can also be replicated using standard sovereign bonds via a suitable dynamic hedging strategy (see Mantilla-Garcia et al., 2019). Hence, in practice, \( B_t \) would be the (rebased) share price of an investable proxy of the corresponding retirement bond. Note that \( B_t \) would be the price of the retirement bond for the cohort of people with target retirement date \( T \), and there would be a retirement bond per cohort.

When the retirement date arrives, and the affiliate needs to start receiving replacement income from their retirement account, the accumulated assets are exchanged for a stream of cashflows. Different types of decumulation products exist, but as discussed above, the price of a retirement bond provides a good proxy for the exchange rate of assets to income if the product is similar to a life annuity contract. Furthermore, if the individual already has part of its retirement assets annuitized (hence having some protection against longevity), they could use the retirement bond as the decumulation product to secure a known level of income during the life expectancy of their cohort. Although the bond does not hedge extra longevity risk, the fact that it remains liquid even throughout the payout phase has at least two important advantages relative to annuity contracts: providing flexibility in case of liquidity needs (related to health issues for instance) and serving bequest motives.
Remark 1 If the retirement bond is the decumulation product used, then the price of these bonds at any date during the accumulation period, \( t \leq T \), constitutes an exact exchange rate of accumulated assets for a known level of retirement income benefits. This is simply because with the current retirement assets \( A_t \), they could buy \( \tilde{N}_t = \frac{A_t}{B_t} \) retirement bonds. Without any further transactions, they will receive \( \tilde{N}_t \) indexed dollars per year with certainty from \( T + 1 \) until \( T + M \). For this reason, we use interchangeably the terms retirement bond and retirement (benefits) unit.

Definition 1 (Securable retirement income \( \tilde{N}_t \) and fund value in retirement units)
If the cost of acquiring one retirement unit at time \( \text{"} t \text{"} \) is \( B_t \), then the value of a portfolio in terms of retirement units is denoted:
\[
\tilde{N}_t := \frac{A_t}{B_t}
\]
(1)
The latter also constitutes the number of real dollars of income throughout retirement that the individual could secure at any time \( t \leq T \) during the accumulation phase, by selling all assets and using the proceeds to buy \( \tilde{N}_t \) retirement bonds. The bonds to be used for this valuation, are the ones that start paying cashflows at the target retirement date \( T \) of the affiliate.

In Appendix B we show that the value of \( \tilde{N}_t \) fluctuates proportionally to the relative performance of the assets in the fund with respect to the retirement bond, plus the extra contributions to the fund (the latter measured in retirement units).

Measuring Liabilities and Funded Status of DC Plans

The liabilities of DB pension plans are usually defined as the present value of the replacement income cash-flows, i.e., the retirement benefits.\(^7\) The latter is a periodic (e.g., annual or monthly) pension payment proportional to the employee’s late-career salary and the years of contribution. For instance, in a typical DB plan for U.S. state employees, “an active worker accrues the right to a periodic benefit upon retirement that equals a flat percentage of his final (or late-career) salary times his years of service with the employer” (Novy-Marx and Rauh, 2011). This type of liability definition will inevitably create a shortfall and eventually require contributions from the plan sponsor. The pension benefit depends on contribution time and the\(^7\)From an economic standpoint, Novy-Marx (2013) established that the correct discount rates to value pension liabilities of DB plans are risk-free yields, not the expected return of the pension fund assets. Furthermore, he shows that using the latter instead of risk-free rates introduces perversive incentives for the fund manager.
latter only has a very loose relationship with the level of contribution dollars needed to finance the increase in the promised level of benefits.\(^8\)

On the other hand, standard DC plans do not have any commitment in terms of retirement benefits. That is, the level of retirement income, i.e., \(\bar{N}_T\), remains unknown until the actual retirement date \(T\). Hence, while there is no financial risk for the pension fund manager, regardless of the impact of their investment decisions, the affiliates bear the risk of ending up with insufficient replacement income, and they only learn this upon retirement (at which points they cannot do much about it). This lack of “liability” or commitment in DC plans leaves affiliates without any level of retirement security. Moreover, it constitutes a major misalignment in risk exposures between the fund management company and the fund affiliates, and a disconnection between the agent’s incentives, and the retirement income objectives of the principal. In order to address this central problem of DC plans, we propose to introduce a “liability” or commitment, defined in terms of a defined level of retirement income.

Before doing that, let us argue that a meaningful and relevant measure of liabilities for a pension fund should be both observable and objective. These two requirements prevent us from using aggregate target levels of replacement income added across individuals as a liability measure. Indeed such targets are not observable and are subject to subjective views of the beneficiaries. Furthermore, unlike DB plan definitions, the liabilities or benefits defined should directly connect to the contributions, so that they are “hedgeable,” meaning that a risk-less investment strategy designed to hedge the liability avoids creating any deficits.

With this in mind, let us now imagine for a moment that a DC pension plan invests all contributions in retirement bonds (corresponding to the retirement date of each participant’s cohort). Note that such a risk-free investment policy is not necessarily optimal since beneficiaries may wish/need to reach target levels of replacement income that may not be affordable in the absence of risk taking. Now let us define the liabilities of a DC plan as the pension cash-flows that would have been secured if all past and present contributions had precisely been invested in retirement bonds. Note that such pension cash-flows are not “liabilities” in a legal sense, and DC plans have not committed to delivering them to beneficiaries. We instead see these liabilities as a natural benchmark against which can be measured the value-added by the

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\(^8\)For example, if the benefit factor is 2% and an employee has worked for 30 years and has an average wage in the last several years of work equal to $35,000, the employee will be entitled to a pension of $24,000 (= 2% * 30 * $35,000) per annum when she retires, plus any COLAs her plan offers. Her guaranteed replacement rate is then 60% = 2% * 30 and the liability of that DB pension plan is the present value of those projected cashflows.
investment decisions of the pension fund manager.

**Definition 2 (Affordable retirement income \( N_t \))** Let \( C_t \) be the value of the contribution to the pension fund at time \( t \), and \( n_t = \frac{C_t}{B_t} \) the number of retirement bonds that can be afforded with that contribution. If all the periodic contributions until time \( t \) were used to buy retirement bonds, then the number of real dollars of replacement income that the affiliate could afford without taking any risk, by any date during the accumulation phase \( t \leq T \) is:

\[
N_t = \sum_{i=1}^{t} n_i
\]

**Definition 3 (Liability for DC plans in current dollars \( L_t \))** Since \( N_t \) is the level of retirement income benefit affordable for the affiliate without taking any risk, by investing all the contributions until time \( t \) in retirement bonds, the liability of the pension fund at any time \( t \) during the accumulation period is defined as the present value of the cash-flows that pay with certainty \( N_t \) retirement bonds:

\[
L_t := N_t B_t
\]

which is equal to the current price of \( N_t \) retirement bonds.

**Remark 2** The liability from Definition 3 measured in retirement units is equal to \( N_t \).

**Remark 3** The number \( L_t \) also represents the dollar value of a fund in which all contributions have been invested in retirement bonds. At the plan level, the global liability portfolio can then simply be obtained via aggregation of individual liability portfolios.

Once a formal liability portfolio process \( L_t \) is obtained, the corresponding funding ratio, and the surplus/shortfall, can be defined similarly to defined-benefit pension plans.

**Definition 4 (Funding Ratio \( FR_t \) and Surplus \( S_t \) for DC plans)** Once a formal liability process \( L_t \) is obtained from Definition 3, the corresponding funding ratio is simply given by:

\[
FR_t := \frac{A_t}{L_t} = \frac{\tilde{N}_t}{N_t}
\]

and the Surplus or Shortfall (depending on the sign) in securable retirement income is:

\[
S_t := \tilde{N}_t - N_t
\]
Note that the securable level of retirement income, $\tilde{N}_t$, mixes the responsibility of the pension fund manager (i.e., the investment strategy), with the contributions to the fund. In that sense, a more accurate metric of investment performance is the funding ratio, as the contributions appear in both the numerator and denominator, and hence their effect is mostly neutralized. Moreover, in Appendix B we show that the dynamics of $S_t$, do not depend on the contributions, but only on the relative performance of the fund:

$$\Delta S_t = \Delta \tilde{N}_t - \Delta N_t = \tilde{n}_{t+1} - n_{t+1} = \tilde{N}_t r_{t+1}^Z$$ (3)

where $r_{t+1}^Z = \frac{1 + r_{t+1}^A}{1 + r_{t+1}^B} - 1$, is the relative returns of the investments in the pension fund (see Eq. 24 and 28 in Appendix B).

In general, investment decisions will differ from a full investment in risk-free retirement bonds, and the relative performance of a given investment strategy with respect to the risk-free retirement bond portfolio can be naturally summarized in terms of the previously defined funding ratio. A ratio greater than 1 means that purchasing power in terms of replacement income has increased since the beginning of the period (relative to the riskless alternative), and a ratio lower than 1 that it has decreased. In other words, an investment strategy is successful ex-post if it leads to an increase in replacement income, that is, if it outperforms the retirement bond liability portfolio over a given period.

Note that if the first date $t_0$ at which the pension liability is estimated is posterior to the creation of the plan, $t_0$ can be taken as the starting date of a fresh risk management perspective and treat $A_{t_0}$ as if it was an initial contribution made at the start of a new pension plan. In that case, since $A_{t_0} = C_{t_0}$, then the initial funding ratio given the Definition 3, is $FR_{t_0} = \frac{A_{t_0}}{L_{t_0}} = 1$.\(^9\)

### 3 Retirement Income Incentives and Target Income Strategies

Using the hedgeable liability definition for DC plans introduced in Section 2, it is possible to design different types of incentives for pension fund management companies in DC plans, to manage assets during the accumulation phase in ways that limit losses of securable retirement income, while pursuing the aspirational goal of a higher income, beyond the level affordable risk-free (i.e., $N_T$).

\(^9\)To see this, note $n_{t_0} = \frac{C_{t_0}}{L_{t_0}}$, and hence $L_{t_0} = \frac{C_{t_0}}{n_{t_0}} B_{t_0} = C_{t_0}$. An alternative approach would be to perform ex-post backfilling starting from the origin of the pension plan as long as a schedule of past contributions is available.
In particular, hereafter we propose a class of liability-driven payment functions that align the interests and the risk exposures of the pension fund management companies with the beneficiaries. Furthermore, they foster retirement security as they embed a strong incentive for managers to secure a level of retirement income that increases with every new contribution:

\[ \tilde{N}_t \geq \kappa_t N_t \quad \forall t \]  

(4)

where \( 0 \leq \kappa_t \leq 1 \) is the proportion to secure. Notice that the liability-driven goal (4) is equivalent to maintaining a minimum level of the DC funding ratio (Definition 4), of:

\[ FR_t = \frac{\tilde{N}_t}{N_t} \geq \kappa_t \quad \forall t \]  

(5)

In Section (3.2), we discuss several alternatives to set the protection proportion \( \kappa_t \), which are motivated by conclusions from the literature of optimal life-cycle investing, or behavioral arguments. Note that the minimum level of income \( \kappa_t N_t \), provides a higher level of understanding and control to the fund affiliates, as they cannot only observe but also take action to increase their minimum level of securable pension benefits.

### 3.1 Liability-driven payment functions

One of the main determinants of the investment strategies followed by fund managers is the type of fee structure of their payment contracts.\(^{10}\) Theoretically, a well-designed incentive scheme should depend on a performance measure that is informative about the agent’s actions, and aligns with the principal’s objective (Baker, 1992). However, it is difficult to find a measure satisfying both conditions in several contexts. It is more common to find so-called “broad” measures that the agents cannot control/affect well, but that induce high alignment of preferences, or “narrow” measures that the agents control well but imply low alignment of preferences (Hall, 2015). If a measure is not informative about the agent’s actions, that means the agent has low control over the observed outputs that the principal receives/observes, and also has high risk exposure to the volatility of the performance measure. That results in weak incentives for the agent. On the other hand, if the performance measure is not aligned with the principal’s objectives, the agent’s effort is not useful for the principal, and the incentives are distorted.\(^{10}\)

\(^{10}\)As a consequence, fee structures for pension fund management companies are regulated in many countries (see Tuesta, 2014).
As previously mentioned, the pension liability of DB plans is unhedgeable, meaning that the managers is unable to prevent funding deficits. Hence, DB funding ratios tend to be broad measures of performance, because the agent has very limited control over the output, and a high risk exposure to its variability. On the other hand, management fees in DC plans usually have the same structure used by regular asset management companies, i.e., a fixed percentage of year-end asset levels:\(^{11}\)

\[
Q^b_t = b A_t \quad \text{with} \quad 0 < b < 1
\]  

(6)

We argue that the payment function (6) is inadequate in the context of pension plans for the following reasons. First, the performance measure (assets), is not in the same units of the principal’s benefits, which results in distorted incentives for the manager. Indeed, if the asset value in a given year increases, but at the same time the “exchange rate” of assets to retirement income \(1/B_t\) decreases by a larger percentage, the payment for the manager would increase, even though the affiliate has experienced a loss in terms of securable retirement income \(\tilde{N}_t\).

Hence, the payment function (6) produces distorted incentives, as it induces an asset-only focus on pension fund strategies, instead of measuring performance in terms of retirement benefits. In other words, pension assets constitute a narrow performance measure.\(^{12}\) Second, unlike regular investment funds, in pension funds, the investor (principal) receives benefits from the fund only after their retirement date, and spread over about 20 years after that date. In other words, there is a large mismatch in the benefit schedules of the principal and the agent (the fund manager) in pension funds, which does not exist in the delegated portfolio management arrangement of regular investment funds.

Note that the agent and principal’s benefit units mismatch is not resolved by paying the agent a number of retirement bonds \(b\tilde{N}_t\), because in principle the agent can sell them at the market price \(B_t\), in which case his payment in current dollars is effectively \(bA_t\) (i.e., back to the payment function 6). This constitutes another example of the irrelevance result (see Admati and Pfleiderer, 1997; Stoughton, 1993). One solution to irrelevance situations is to restrict the manager from undoing, with transactions in their personal portfolios, the incentives given by

\(^{11}\) Another standard payment structure for pension fund managers is to charge a fixed percentage of the contributions to retirement accounts (see Han and Staňko, 2020). The latter fee type has the problems of both, the broad and the narrow metrics because, on the one hand it does not incentivize efforts to improve retirement income benefits for affiliates, as it is the principal, not the manager, that controls contributions (when the latter are not mandatory), and it also has the problem of being delinked from the level of pension benefits.

\(^{12}\) Note that for mandatory pension plans, the assets value as a performance metric, also has the flaw of broad measures, because \(A_t\) depends to a large extent, not only on the asset returns, but on the series of mandatory contributions \(C_t\) for all \(t\).
the payment function. In this case, one could try restricting the agent (e.g., by regulation) from selling the retirement bonds $b\tilde{N}_t$, so that the agent’s payment schedules would match with the principal’s one. However, notice that while the units and schedules align under the selling restriction, the risk exposures do not. To see this, note that the number of retirement bonds (hence its future income) that the agent accrues during the principal’s accumulation period, is strictly increasing. On the other hand, the principal’s expected future retirement income, which is equal to $\tilde{N}_t$, fluctuates all the time during the accumulation phase. Hence, it would be possible that after paying many retirement bonds to the agent, the principal losses many more if the value of $\tilde{N}_t$ decreases (either due to asset losses, or increases in $B_t$). Furthermore, given the “forced” position in retirement bonds that the manager holds, the latter would have a strong incentive to take on more risk (most likely, beyond what would be optimal for the principal). An alternative to solve this misalignment in risk exposures, would be to pay the manager only after retirement date a number of retirement bonds equal to

$$Q_t = \begin{cases} 
  b \tilde{N}_T & \text{for } t = T \\
  0 & \text{for all } t < T 
\end{cases} \quad (7)$$

and to restrict the manager from selling those bonds or undoing this position in the market with other types of operations. In that case, the cashflows received by the pension fund management company would be a proportion $b$ of the total retirement income that the beneficiaries would receive in the absence of fees, and on the same dates. However, the payment function (7) has a few caveats. First, in practice it can be very difficult to prevent the manager from undoing their position, because even if a regulation prevents the pension fund management company from selling the retirement bonds, the position can also be undone by taking loans matching the bonds cashflows (and such loans could be taken by the shareholders of the management company). Second, the fact that the payment function (7) requires to fix the payment (in number of retirement bonds) only upon the retirement date, constitutes a barrier or disincentive for management to enter the contract, particularly as fund affiliates are usually free to change pension providers during the accumulation period. Third, note $\tilde{N}_t$ increases with fund contributions, and in most countries, the contributions to pension funds are mandatory or semi-automated (usually being a proportion of labor income). As a result, the metric provides weak incentives to make efforts, to the manager, as the contributions do not depend on the decisions taken by the manager.
To address the problems discussed above, we propose the following payment function:

\[ Q_t^8 = b (A_t - \kappa L_t) \quad \text{with} \quad 0 < \kappa < 1 \quad ; \quad 0 < b < 1 \]  

(8)

Note that the payments for the management company are not delayed in this case (they can be done year-end during the accumulation period, as in current practice). Besides, several other remarks are in order. First, note that the performance measure that drives the incentives is the risk-adjusted surplus in pension benefits (Detemple and Rindisbacher, 2008, refer to a quantity similar to the risk-adjusted surplus as excess coverage). Indeed, if the level of securable retirement income \( \tilde{N}_t \) increases (decreases) on a given year, the payment for the manager also increases (decreases). To see this, express Eq. (8) in number of retirement bonds, i.e., \( Q_t = Q^8_t / B_t \),

\[ Q_t = b (\tilde{N}_t - \kappa N_t) \]  

(9)

Hence, it aligns the performance measure of the manager, with the benefits of the principal. Second, note that the risk-adjusted surplus is measured relative to a hedgeable liability measure, and is less dependent in the contributions, which means that the fund manager has more control over the outputs relative to the equivalent metric in DB systems, and relative to the assets level, for the reasons previously discussed.\(^{13}\) In that sense, this performance measure is more balanced, i.e., less broad than the excess coverage of DB plans (due to the unhedgeable nature of DB liabilities), and less narrow than the standard payment function (6).

Third, Eq. (9) indicates that if the value of the fund in number of retirement units, \( \tilde{N}_t \), suffers a cumulative loss large enough to trespass below the threshold or floor value \( \kappa N_t \) (and equivalently, if the funding ratio falls below \( \kappa \)), then the fund management company would also suffers a loss, because \( Q_t < 0 \), which represents a penalty to the fund management company. In that sense, payment structure (8) aligns the risk exposures of the pension fund manager company and the beneficiaries.\(^{14}\) However, note that the risk budget implied by the parameter \( \kappa \) plays a crucial role, because if the fee was proportional to the unadjusted surplus, i.e., \( Q_t = b (\tilde{N}_t - N_t) \), the function could kill the incentives for the fund management company to take risk, as any loss, regardless of how small, would generate penalties already. Indeed, if pension fund management

\( ^{13} \)Note \( A_t \) and \( L_t \) increase by the same amount with each contribution.

\( ^{14} \)The money paid as penalty could be reinvested in the fund to partially compensate the beneficiaries for the loss. This is similar in spirit to the additional contributions that plan sponsors have to make in a DB schemes in cases of severe defunding, albeit with a very fundamental difference: the hedgeable property of the liability in our case, which implies that penalties can be avoided in all cases, as discussed below.
companies have preferences similar to the ones implied by the general class of utility functions proposed in Detemple and Rindisbacher (2008), then their optimal strategy would be to take zero risk.\textsuperscript{15} Hence, using $\kappa < 1$ creates a risk budget, that allows having both, incentives to take risk, but also to control losses.

While the possibility of experiencing penalties (negative fees) could be considered a disincentive to join the contract, in Section 3.2 we show that the pension fund management company can avoid paying any penalties at all times and in every scenario under the proposed payment function (8), i.e., $Q^8_t > 0$ for all $t$, by using what we call a target income strategy, which secures a strictly increasing level of retirement income, i.e., $\tilde{N}_t \geq \kappa N_t$.

Fourth, the proposed payment function Eq. (8) is flexible enough to integrate other types of incentives that are motivated by the literature of optimal life-cycle investing, or behavioral arguments, using a time-varying risk-budget through the parameter $\kappa_t$, and without losing the desirable properties discussed in this section. In particular, it can include an incentive to decrease risk-taking as the retirement date approaches, or an incentive to protect part of cumulative gains (a ratchet effect), or a combination of the two, by defining $\kappa_t$ as described in Section 3.2.

3.2 Target income strategies

The payment function proposed in Eq. (9) gives a strong incentive for the fund manager to maintain the level of securable income benefits for its affiliates above a fraction $\kappa_t$ of the affordable level $N_t$. Given an investable alternative for $B_t$, i.e., the retirement bond or a liability-hedging portfolio that replicates its value, it is straightforward for the pension fund management company to secure that increasing level of retirement income for its affiliates, while accessing part of the upside potential of risky assets, by investing a proportion of capital in the risky performance-seeking portfolio (PSP):\textsuperscript{16}

$$\omega^{PSP}_t = \frac{\tilde{N}_t - \kappa_t N_t}{\tilde{N}_t} = \frac{A_t - \kappa_t L_t}{A_t}$$

and investing the remaining proportion of capital, i.e., $\omega^{LHP}_t = 1 - \omega^{PSP}_t$, in retirement bonds (or in shares of the corresponding replicating fund). This rule implies holding a number of

\textsuperscript{15}The class of utility functions in Detemple and Rindisbacher (2008) represent the problem faced by the sponsor of a DB pension plan, which exhibits an infinite disutility for failure to meet payments of the defined benefits (i.e. it is assumed that the fund sponsor perceives a technical default as being extremely costly).

\textsuperscript{16}The second equality follows from the definitions $\tilde{N}_t = \frac{A_t}{B_t}$ and $N_t = \frac{L_t}{B_t}$ (see Eq. 1 and 3).
retirement bonds, at all times, equal to

\[
\omega_t^{LHP} \times \tilde{N}_t = \kappa_t N_t
\]  

(11)

In Appendix C, we show that a sufficient condition for the allocation rule (10)-(11) to ensure that \( \tilde{N}_t \geq \kappa_t N_t \), at every time \( t \) for every possible value for the risky assets’ returns and interest rates levels, is that the dynamics of \( \kappa_t \) satisfies:

\[
\Delta \kappa_t \leq (1 - \kappa_t) \frac{N_{t+1}}{N_t}
\]  

(12)

where \( \Delta \kappa_t = \kappa_{t+1} - \kappa_t \). Hence, even in the extreme event that the risky PSP has a return of \(-100\%\), following the simple allocation rule (10)-(11) with a \( \kappa_t \) process that satisfies condition (12), guarantees that the portfolio’s value remains above its floor, \( \kappa_t N_t \), and the funding ratio is always above the level \( \kappa_t \).

We now present four different alternatives for \( \kappa_t \), which are further characterized and compared with a typical target date fund strategy through historical simulations in Section 4.2. However, notice that Eq. (10)-(11) together with condition (12) define a more general class of target income strategies, as \( \kappa_t \) could take other forms as long as it respects condition (12).

The simplest target income strategy uses a constant risk budget, i.e., \( \kappa_t = \kappa \) at all times. In that case, for \( \kappa < 1 \), it satisfies the condition (12) because \( \Delta \kappa_t = 0 \). We refer to this strategy in short as \( FR \), as it ensures that the funding ratio always stays above a constant predefined proportion \( \kappa \). Investors with higher loss aversion would prefer strategies with a higher protection level \( \kappa \).

Instead of keeping a constant risk budget parameter throughout the accumulation period, we propose using an increasing proportion \( \kappa_t \) of the affordable retirement income \( N_t \), so as to take more risk when the investors are younger, and securing an increasingly higher level of replacement income as their retirement date approaches. This follows a similar logic to the asset allocation dynamic of target date funds, but decreasing risk as measured in number of retirement income dollars, instead of focusing on the variations in assets only. In Section 4.2 we show that the change of measurement units from current dollars to \( \tilde{N}_t \), and using retirement bonds instead of short-term bonds as the safe asset, constitutes a very large difference. In
particular, we propose defining the proportion $\kappa_t$ in (10)-(11) as:

$$\kappa_{CFR}^T = \left( 1 - (1 - \kappa) \frac{\hat{N}_T}{N_t} \right)^+$$

(13)

with $0 < \kappa < 1$. The proportion $\kappa_{CFR}^T$ never decreases, and increases (only) with every new contribution $n_t$. Hence, if there are frequent contributions, the protected proportion increases as the retirement date approaches. In Appendix D, we show that $\kappa_{CFR}^T$ satisfies the condition (12) with equality, hence the floor income process $\kappa_{CFR}^T N_t$ can also be secured following the allocation rule (10)-(11).

The parameter $\hat{N}_T$ is a target level of affordable retirement income, and is also the target level of contribution in number of retirement bonds. This level should be set taking into account the characteristics of each individual, particularly age and expected income. In the empirical simulation in Section 4.2, we set it as the expected level of income before retirement, based on the average age-income profiles per education level, constructed from data for average labor income series for workers in the United States (U.S.) for age-income profiles per education level based on the Panel Study of Income Dynamics (PSID), as in Cocco et al. (2005).

In general, the actual level of total contributions that will be reached by the retirement date, i.e., $N_T$, is unknown. This is particularly true if the contributions are defined as a fixed percentage of labor income (which is often the case in DC plans). From the definition Eq. (13), it follows that:

$$\kappa_{CFR}^T = \begin{cases} \kappa, & \text{if } N_T = \hat{N}_T \\ \kappa < \kappa_{CFR}^T < 1, & \text{if } N_T > \hat{N}_T \\ 0 < \kappa_{CFR}^T < \kappa, & \text{if } N_T < \hat{N}_T \end{cases}$$

(14)

Furthermore, the difference between the minimum retirement income insured by the strategy and the target floor level is

$$\kappa_{CFR}^T N_T - \kappa \hat{N}_T = N_T - \hat{N}_T$$

Hence, if an overshoot in contributions occurs, relative to the target level, i.e., $N_T > \hat{N}_T$, then the minimum level of retirement income that the strategy secures, would be higher than the target floor level, i.e., $\kappa_{CFR}^T N_T > \kappa N_T > \kappa \hat{N}_T$. Conversely, if an undershoot in contributions

---

In principle, it is possible to vary the percentage of labor income contributed to the retirement account, such that a given path of predefined contributions that sum to $\hat{N}_T$, is met at each point in time. However, we focus here on the case of current contribution paths in DC plans.
occurs, a proportion lower than $\kappa$ of $N_T$ is insured. However, that does not necessarily result in a lower level of retirement income. In fact, in those cases, the strategy takes more risk by allocating more to the performance-seeking portfolio, which may increase the changes of achieving an income level of $\tilde{N}_T$ (or even higher). However, the resulting level of retirement income in the most unfavorable scenarios, would be lower.\footnote{In practice, a possible modification of the strategy, is to make revisions of the target income level $\hat{N}_T$, depending on whether the expected overshoot or undershoot is too large, as the uncertainty about the contributions resolves over time. While securing a lower proportion of $\tilde{N}_t$, which implies a higher risk exposure, can increase the chances of achieving the target level of retirement income $\tilde{N}_T$ or even higher, it also increases the chances of a larger shortfall. Indeed, depending on the level of risk aversion of the investor, the expected surplus might not compensate the extra level of risk taken. Conversely, if the contributions grow past the initially planned target $\tilde{N}_T$, the proportion of affordable income that is secured is higher than $\kappa$. In that case, the investor may prefer to take more risk, by revising upwards the target level $\tilde{N}_T$.}

Another common objective in fund management, is to protect a proportion of past cumulative gains (see for instance Lan et al., 2013). Hence, we also consider a strategy that secures a proportion $\kappa$ of the maximum level of retirement income attained, at a previous time $t^* \leq t$, plus the same proportion of all the contributions made to the fund since time $t^*$, i.e.,

$$\tilde{N}_t \geq \kappa \tilde{N}_{t^*} + \kappa \sum_{i=t^*+1}^{t} n_i$$  \hspace{1cm} (15)$$

where $t^*$ is the latest time at which the surplus $S_t = \tilde{N}_t - N_t$ reached its running maximum for all the observations until time $t$. Note that the insured level of retirement income (15) never decreases and increases with every extra contribution and in periods of good relative performance of the risk assets in the fund with respect to retirement bonds (i.e., whenever a new maximum $\tilde{N}_{t^*}$ is attained). Also note that this protected level $\kappa \tilde{N}_{t^*} + \kappa \sum_{i=t^*+1}^{t} n_i \geq \kappa N_t$ at all times by definition of $t^*$.

Rearranging terms in Eq. (15), the minimum income level in the rhs of Eq. (15) can be expressed as a special case of (4), with a proportion $\kappa_i$ of the affordable retirement income $N_t$ equal to\footnote{To see this, note that $\sum_{i=t^*+1}^{t} n_i = \tilde{N}_t - N_{t^*}$.}

$$\kappa_i^{FR^*} = \kappa \left(1 + \frac{S_{t^*}}{N_t}\right)$$  \hspace{1cm} (16)$$

with $0 < \kappa < 1$, where $S_{t^*}$ denotes the maximum level of surplus observed until current time $t$. The proportion of $\kappa_i^{FR^*}$ increases each time a new maximum is attained in the level of surplus. If there are extra contributions before a new maximum surplus level is attained, the proportion decreases with each contribution, but the minimum protected level of retirement income never decreases (see Eq. 15). Also, note that by definition, at the initial date $\tilde{N}_0 = N_0$, thus $S_{t^*}$ is
always non-negative, hence $\kappa_t^{FR^*} \geq \kappa$ at all times. This means that the target income strategy (10)-(11) with $\kappa_t = \kappa_t^{FR^*}$ is a more conservative strategy than the one with $\kappa_t = \kappa$. Also, since $S_{t^*}$ is expected to grow, everything else equal, the equity allocation of the strategy $FR^*$ decreases over time relative to the allocation of strategy $FR$. In Appendix E we show that $\kappa_t^{FR^*}$ satisfies the condition (12), hence the floor income process $\kappa_t^{FR^*} N_t$ can also be secured following the allocation rule (10)-(11).

We finally propose a fourth type of target income strategy that combines the horizon-derisking mechanism in $\kappa_t^{CFR^*}$, with the ratchet effect on cumulative gains of retirement income of $\kappa_t^{FR^*}$, by securing at all times a proportion of $N_t$ equal to

$$\kappa_t^{CFR^*} = \left( \kappa_t + \kappa \left( \frac{S_{t^*}}{N_t} \right) \right)^+ \quad (17)$$

where $\kappa_t^C = 1 - (1 - \kappa) \frac{\tilde{N}_t}{N_t}$ and $S_{t^*}$ is the maximum surplus attained by time $t$. Note that when the protected proportion $\kappa_t^{CFR^*} > 0$, the secured income floor can be decomposed as a strictly increasing proportion $\kappa_t^C$ of $N_t$ plus a proportion $\kappa$ of the maximum previously attained surplus in retirement income, i.e.,

$$\tilde{N}_t \geq \kappa_t^C N_t + \kappa S_{t^*} \quad (18)$$

In Appendix F we show that $\kappa_t^{CFR^*}$ satisfies the condition (12), hence the floor income process $\kappa_t^{CFR^*} N_t$ can also be secured following the allocation rule (10)-(11). Also note that, because at the initial date $\tilde{N}_0 = N_0$, hence $S_{t^*}$ is always non-negative, which implies that $\kappa_t^{CFR^*} \geq \kappa_t^C$ at all times. This means that the target income strategy (10)-(11) with $\kappa_t = \kappa_t^{CFR^*}$ is a more conservative strategy than the one with $\kappa_t = \kappa_t^{CFR}$. Also, since $S_{t^*}$ is expected to grow, everything else equal, the equity allocation of the strategy $CFR^*$ decreases faster than the allocation of strategy $CFR$.

An attractive property of the strategies with the ratchet effect on surplus ($FR^*, CFR^*$), regards the dynamics of their funding ratio. In Appendix G we show that, in periods in which there are no contributions from $t^*$ until current time $t$, the max drawdown (MDD) in funding ratio of strategy $FR^*$ is bounded by $\kappa - 1$. For instance, if $\kappa = 0.7$, any percentage change observed in the funding ratio relative to its running maximum $FR_{t^*}$, is bounded from below at $\kappa - 1 = -30\%$. Furthermore, we show that the MDD in funding ratio of strategy $CFR^*$ is also bounded, and the lower limit on cumulative returns increases with the level of contributions, and hence the maximum losses decrease as retirement approaches. This limit converges to the
same bound, $\kappa - 1$, when the contribution level at time $t^*$ (the time when the maximum funding ratio was reached), has reached the target level $\hat{N}_T$ (see Appendix G for details).

Regarding the strategies with horizon derisking, the increasing threshold parameter $\kappa_t^{CFR}$ and $\kappa_t^{CFR*}$ decreases the exposure to the PSP, as the retirement date approaches. For instance, in the empirical simulation presented in Section 4.2, the equity allocation of these two strategies starts at 100\% at the beginning of the accumulation period, decays at a relatively high rate, and then stabilizes as the retirement approaches (see Figure 5). Interestingly, this same pattern is observed in the optimal allocation strategy proposed in Levy and Levy (2021) (see the bottom panel in Figure 4 in that paper). However, unlike the target income strategies introduced here, Levy and Levy (2021) focuses on absolute wealth at retirement date, instead of the level of income benefits throughout retirement.

The target income strategies not only provide retirement income security, but also generate helpful and clear signals to affiliates for their savings/consumption decisions during the accumulation period. Indeed, the minimum level of retirement income that has been secured with all previous contributions $\kappa_t N_t$, is a piece of information that almost any adult understands, because its units are income dollars, which is perhaps the most familiar financial figure for everyone, as it is needed to budget current consumption.\textsuperscript{20} Furthermore, that minimum level of secured income never decreases, regardless of the movements in interest rates and equity returns, and increases with every new contribution to the retirement account.\textsuperscript{21} This means that during the years prior to the retirement date, each affiliate knows what is the minimum level of income benefits to expect, hence no ‘last minute’ disappointments occur when the accumulated assets are converted to retirement income (in fact, we believe that reporting the level of assets is a misleading number). It also means, that affiliate involvement might increase, as they can also observe the direct impact of an extra contribution, in the minimum income level that they can secure.

To this last point, note that for the strategies with horizon derisking ($CFR$ and $CFR^*$), the income floor $\kappa_t N_t$ can be expressed in a couple of alternative ways that can be useful for affiliates to understand how they are progressing toward the target income level $\hat{N}_T$, and relative to the target level of protection $\kappa$ (instead of the increasing but lower $\kappa_t$). In particular, the

\textsuperscript{20}Recall that such income figure will grow automatically over time with the COLA adjustment embedded in the retirement bond.
\textsuperscript{21}For the two strategies with ratchet effect (i.e., $FR^*$ and $CFR^*$), the floor value $\kappa_t N_t$ increases with every contribution and whenever a new maximum surplus level is attained.
income floor of CFR* (Eq. 18) is equivalent to:

\[
\hat{N}_t \geq \kappa \hat{N}_T + \kappa \hat{N}_T^* - (\hat{N}_T - N_t) \\
\tilde{N}_t \geq \kappa \tilde{N}_t^* + \kappa (N_t - N_t^*) - (1 - \kappa)(\hat{N}_T - N_t)
\]

Thus, if the contributions reach the target level, her minimum replacement income is $\kappa \hat{N}_T + \kappa \hat{N}_T^*$. On the other hand, if the affiliate does not make any contributions from $t$ until retirement date $T$, the minimum replacement income benefit to be received is $\kappa \hat{N}_T + \kappa \hat{N}_T^* - (\hat{N}_T - N_t)$ (plus COLAs). Also, if the affiliate contributes only a percentage $1 - \kappa$ of the planned contributions, in other words the total contributions done after $t$ are equal to $(1 - \kappa)(\hat{N}_T - N_t)$, then her minimum replacement income at time $T$ will be $\kappa \hat{N}_t^* + \kappa (N_t - N_t^*)$.\textsuperscript{22} The previous income figures, can also be useful information for affiliates deciding whether to make or not extra contributions at any moment during the accumulation phase. For instance, if they decided to make an extra contribution of $(1 - \kappa)(\hat{N}_T - N_t)$, they would be securing a level of income in retirement of $\kappa \tilde{N}_t^* + \kappa (N_t - N_t^*)$.

4 Empirical Analysis

4.1 Measuring assets’ risk in retirement units

Hereafter we present the result of historical simulations of the target income strategies introduced in the previous section and a comparison with the strategy followed by many pension funds in the United States and other countries referred to as target date funds (TDF). The asset allocation strategy of TDFs commonly consists of allocating to an equity fund 100 minus the investor’s age (as a percentage of assets). We simulate the performance of cohorts of investors that contribute to their retirement account during 35 years and assume a retirement age of 65 years. Hence, the investor starts to contribute to her retirement account at age 30, and the initial allocation to equities is 100-30 = 70 percent. The portfolio is rebalanced to have an allocation to equities that decreases linearly every month until it reaches 35% at the retirement age of 65. TDF’s commonly allocate the rest of the capital to a bond index fund with relatively low duration, e.g., 5 years approximately\textsuperscript{23}. The reason is that these bond portfolios have a

\textsuperscript{22}Note that $(N_t - N_t^*) \geq 0$ at all times, because $t \geq t^*$ and $N_t$ is a non-decreasing quantity by definition.

\textsuperscript{23}For instance, the Duration of the Bloomberg Barclays US Aggregate Bond Index during the period 1978-01-01 to 2019-09-30 presented a mean-reverting behavior around an average value of 4.9 years. This bond index is used as the bond benchmark by Target-Date funds, such as the LifePath Index 2055 Fund of BlackRock (they
relatively low short-term volatility, hence (mistakenly) considered low-risk. As we illustrate hereafter, in terms of retirement income dollars, these assets are as risky as an equity index due to the large duration-mismatch they have with the retirement cashflows that the portfolios aim to finance during the payout phase. Hence, in terms of expected retirement benefits, the TDFs are not using an asset that allows the strategy to decrease risk effectively, as the retirement date approaches.

To illustrate this, we use historical data for US government bond yields and the Nelson and Siegel (1987) model to simulate the returns of a bond index with a constant duration of 5 years and returns of the retirement bonds described in Appendix A. Nominal bond yields are available since 1962 for several maturities, while real bond yields have shorter history and less maturities are issued. We simulate nominal retirement bonds instead of real ones to have a more complete empirical analysis, with relatively long periods during which interest rates increase and periods during which interest rates decrease significantly. The retirement bonds simulated pay a series of nominal cashflows growing at a fixed COLA rate of 2% per year.

Several authors have reported ‘numerical difficulties’ estimating the parameters of the Nelson-Siegel model. In order to avoid parameter estimation issues, we follow the ‘data-based’ definitions in Diebold and Li (2006), with the level factor proxied as the 10-year yield, the slope as the difference between the 10-year and the short-term yield, and the curvature as the twice the medium-term yield minus the sum of the short-term and 10-year yields. From 1962-01-0 to 1976-05-31, dates during which the 2-year yield (‘TCMNOM_Y2’) is not available in WRDS, we use the 5-year yield (‘TCMNOM_Y5’) as the medium term rate instead and from 1962-01-02 to 1997-01-01, dates for which the CRSP 3-month yield (‘NFCP_M3’) is not available, we use the 1-year yield (‘TCMNOM_Y1’) as the short-term rate. The switch in the yields that we use in the middle of the sample is done solely to have historical simulations that span the periods displaying the most significant movements in government yields, but the resulting time series has no abrupt changes around the switch dates. Figure 1 presents the time series of the resulting Nelson-Siegel model parameters used for the historical simulations.

report $200 billion invested in “LifePath Target Date fund’s assets”).

24The source for the yields data is the database of the Federal Reserve of Saint Louis (https://fred.stlouisfed.org/series).

25For instance, Gilli et al. (2010) show that in certain ranges of the parameters, the model is badly conditioned, thus estimated parameters are unstable given small perturbations of the data. Indeed, the optimization problem for the parameter estimation is not convex and has multiple local optima.

26Indeed, the last 1-year yield used was 5.57% on 1997-01-01 followed by the first 3-month yield used of 5.30% on 1997-01-02. The last 5-year yield used was 7.73% on 1976-05-31 and the first 2-year yield on 1976-06-01 was 7.26%.
The dataset includes monthly time series of the CRSP’s S&P 500 equity index total returns, the Nelson-Siegel parameters series described above from 1962-01-02 to 2019-12-30, and average labor income series for workers in the United States (U.S.) for age-income profiles based on the Panel Study of Income Dynamics (PSID), i.e., the largest longitudinal U.S. dataset containing information on labor income and individual control variables. The methodology for the age-income profiles follows Cocco et al. (2005). We simulate the retirement accounts for an average individual in each of all the possible cohorts in the sample during their accumulation phase, assuming that each individual makes monthly contributions during 35 years until retirement. The monthly contributions are set to 12.4% of the estimated monthly income, which is the mandatory contribution in the U.S. (OECD, 2017). The real monthly income series is estimated as the annual income numbers from the age-income profiles divided by 12. The average income profile presents an inverted u-shape, reaching a maximum level by age 46 of 38.6k per year, and decrease to 26.38k by age 65 (see Figure 1 in Cocco et al., 2005).

We simulate the behavior of 277 retirement accounts, which is the number of months from 1996-12-31 (35 years after the sample’s start) until 2019-12-30. For each cohort, we compare the hypothetical historical performance of the TDF with the four target income strategies introduced in Section 3.2.

For illustration purposes, Figure 2 displays the cumulative return of the equity index, the constant duration bond index and the retirement bond for two of the simulated periods. The left Panel corresponds to the simulation of the cohort retiring in 1996-12-31 (first cohort in the sample), which experiences a significant increase in interest rates levels, and the right Panel to the cohort retiring in 2019-12-31 (last cohort), which observes a large decrease in the level of interest rates. In the former period, the value of the equity and short-duration indices presents a larger increase than the retirement bond of the cohort, while for the latter period, the opposite occurs.

More generally, Table 1 presents summary statistics of the returns for the equity index, the constant 5-year duration bond index and each of the 277 retirement bonds. The median annualized return in the sample, across the 277 periods of 35 years each, is 11.32% for the equity index, 8.11% for the constant duration index and 8.68% for the retirement bond. The annualized return for the retirement bonds ranges from 3.1% to 21.2%, compared to 6.2% to 8.8% for the bond index, and 9.5% to 12.4% for the equity index (see Panel A). In particular, the quantiles 75% and above of the annualized return distribution are higher for the retirement
bond than for the equity and bond indices, which indicate that in many scenarios, there is the potential of losses in terms of retirement income dollars, relative to the lowest risk alternative of investing in the corresponding retirement bond. Panel E in Table 1 presents the relative drawdown (RDD) of these two indices with respect to the corresponding retirement bond for each of the 277 sample periods. The RDD is defined as the maximum drawdown of the relative value process. The latter is \( Z_t = \frac{S_t}{B_t} \), where \( S_t \) denotes the asset value, and \( B_t \) is the price of the retirement bond of the respective cohort. Thus, the RDD measures the maximum cumulative loss in number of retirement bonds that an investment in those indices could secure at each point in time during the accumulation phase. Despite the high correlation displayed between the bond index and the retirement bonds (ranging between 0.82 and 0.9, as shown in Panel D), its RDD ranges from 73.9% to 98%, which is very similar and even slightly higher than the RDD of the equity index for most quantiles (the RDD of the equity index ranges from 74.8% to 97.7%, and its correlation with the retirement bonds from 0.05 to 0.28). Hence, the constant-duration bond index is as risky as the equity index, when its variations are measured in units of relevance for pension fund affiliates.

In contrast, if one were to measure the risk of these assets in current dollars during the accumulation phase, instead of doing it in retirement income dollars, the assessment would have been opposite to the previous one, and misleading in the context of a pension fund. For instance, as Panel B of Table 1 shows, while an investment in retirement bonds has by definition zero variations in terms of the number of securable retirement income dollars, its value in current dollars varies significantly during the accumulation phase, and presents significantly higher short-term volatility than the equity and bond indices. The annualized standard deviation of its monthly returns ranges between 31.8% and 49.6% compared to average volatilities of 14.4% to 15.9% for the equity index and 6.1% to 9.3% for the constant-maturity bond index. Nonetheless, investors in a pension fund usually do not have any benefits during the accumulation phase from their retirement accounts, and hence, these variations do not correspond to the relevant units for them, as the asset level alone, is a misleading metric of the expected level of benefits.

### 4.2 Target income strategies vs. target date funds

A widespread rule of thumb to set a target level of replacement income that would approximately sustain living standards after retirement, corresponds to a replacement rate of 70%. Hence, to simulate the four target-income strategies described in the previous section, we set \( \kappa = 0.7 \)
and $\hat{N}_T$ equal to the average labor income at age 65 in real dollars.\textsuperscript{27} In the historical period analyzed, the final level of contributions $N_T$ obtained from investing 12.4% of the average income figures, is more than the target level of contribution $\hat{N}_T$ for 275 out of the 277 cohorts, and the minimum contribution ratio, i.e., the lowest level observed for $\frac{N_T}{\hat{N}_T}$ is 99.3%. As discussed in Eq. (14) in Section 3.2, the effect of an ‘overshoot’ in contributions for the strategies with horizon derisking, relative to the target contribution level $\hat{N}_T$, is securing a proportion of the affordable retirement income above the predefined level $\kappa = 0.7$ by the retirement date, i.e., $\kappa_T > \kappa$. The effect of an ‘undershoot’ on the expected contributions, is that the strategy ends up securing a lower proportion of the affordable retirement income $N_T$. Notice that a lower secured proportion does not necessarily result in a lower level of retirement income. In fact, in those cases, the strategy takes more risk, which may increase the changes of achieving a higher income level $\hat{N}_T$. However, the resulting level of retirement income in the most unfavourable scenarios, would be lower. Table 2 presents a distribution summary of the resulting proportion $\kappa_T$ across the 277 simulated cohorts, and for the age-income profile disaggregated by the three education levels, and for the average income across the three, which is the one used for the strategies simulation. The proportions obtained range between 0.69 and 0.92.

Figure 3 presents the securable level of retirement income, $\tilde{N}_t$, for the TDF, the four target income strategies, and their Floor value, $\kappa_t N_t$, at each point in time for the first cohort (left panels) and last cohort (right panels). The left panels present the simulation of the scenario with increasing interest rates (from January 1962 to December 1996), which represents a favorable scenario for the equity index and the TDF relative to the performance of retirement bond over the 35 year period, while the right panel represents a less favorable for the indices relative to the retirement bond (from January 1990 to December 2019). In the first scenario, the TDF yields a level of retirement income 5% higher than the target income strategy without derisking, $FR$, and 10% higher than $FR^*$, but a lower one than the strategies with an increasing protection level $\kappa_t$. Indeed, the strategies with derisking, $CFR$ and $CFR^*$, yield a level of retirement income 44% and 23% higher than the TDF in that scenario. Furthermore, the TDF displays a much larger level of variations towards the end of the sample, particularly when compared with the strategies without derisking and with the strategy with derisking that includes the ratchet effect, i.e., $CFR^*$, as observed in the left panels of Figure 3.

\textsuperscript{27}The average income at age 65 from the age-income profiles data, across the three education levels is approximately 26.38k per year, or 2198 per month. The base year from the real income data is 1992.
In the less favorable scenario for the equity index (from January 1985 to December 2019), the more protective strategies, i.e., target income without derisking (i.e., $FR$ and $FR^*$) yield an increment in the level of retirement income with respect to the TDF of 75% and 82%, while the strategies with derisking an increment of 56.4% and 56.9%, all of them with a much lower level of variations in $\tilde{N}_t$ than the TDF, as observed in the right panels of Figure 3.

One important advantage of the target income strategies over the TDFs, is that they generate a relevant and clear signal to pension fund affiliates during the accumulation phase: the minimum level of retirement income that has been secured with all previous contributions to the retirement account, at every point in time, $\kappa_t N_t$. Indeed, for the target income strategies introduced, this minimum level of secured income never decreases, regardless of the movements in interest rates and equity returns, and increases with every new contribution to the retirement account.\footnote{For the two strategies with ratchet effect (i.e., $FR^*$ and $CFR^*$), the floor value $\kappa_t N_t$ increases with every contribution but also whenever a new maximum surplus level is attained.} Hence, each pension affiliate knows the level of retirement income that they have secured at every moment during the accumulation phase (a number that will also increase with future COLAs embedded in the retirement bond), which is an instrumental piece of information to take consumption/savings decisions, particularly during the latter part of the accumulation phase. More importantly, this also means that at the retirement date, there are no bad surprises in terms of the benefits received, when the conversion from assets to retirement income occurs. This is because the income level is in every case above the Floor value $\kappa_T N_T$, which will be close and above to $\kappa_{T-1} N_{T-1}$ and all the preceding values for it, which only increase proportionally to the contributions. Such information comes in a form that is perhaps the only financial figure that virtually any adult understands: current income dollars. Indeed, the units of the Floor value process $\kappa_t N_t$ are monthly income dollars in real terms, and the latter can be easily converted to current nominal dollars.\footnote{For instance, if the cashflow paid on the first month by each retirement bond, is a dollar plus a fixed COLA adjustment of 2%, then the floor value $\kappa_t N_t$ can be deflected with that rate to express current dollars.}

In contrast, although the value of TDFs (or any other fund) can be expressed in the number of securable retirement dollars, $\tilde{N}_t$, the latter is very volatile for that kind of strategy, even during the period close to retirement (see the right panels in Figure 3), and it has no minimum level (besides zero).

As discussed in Section 2, the affordable level of retirement income, $\tilde{N}_t$, mixes the responsibility of the pension fund manager (i.e., the investment strategy), with the contributions to

\footnote{For instance, if the cashflow paid on the first month by each retirement bond, is a dollar plus a fixed COLA adjustment of 2%, then the floor value $\kappa_t N_t$ can be deflected with that rate to express current dollars.}
the fund. In that sense, a more accurate metric of investment performance is the funding ratio introduced in Definition 4. Figure 4 presents the funding ratio series of the TDF and the four target income strategies over the aforementioned periods. In the less favorable scenario (right panels), the TDF presents a final funding ratio of 0.57, while the target income without derisking present funding ratios of 1.01 and 1.04, and the strategies with derisking of 0.897 and 0.894. In the more favorable period for equities, the TDF displays a final funding ratio of 1.4, while the target income without derisking presents funding ratios of 1.33 and 1.27, and the strategies with derisking of 2.0 and 1.7.

More generally, Table 3 presents the distribution summary of the funding ratio across the 277 scenarios for the TDF and the four target income strategies. Panel A displays a distribution summary of the resulting funding ratio at the retirement date. The median value for the final funding ratio of the TDF is 0.55, while for the $FR, FR^*$, $CFR$ and $CFR^*$ strategies is 0.94, 1.03, 1.12 and 134, respectively. Moreover, all the reported quantiles of the TDF’s funding ratio distribution are dominated by the corresponding quantiles of the four target income strategies, except for the maximum value of the $FR^*$ strategy. In fact, in 269 out of the 277 scenarios the $FR$ strategy results in a higher final funding ratio than the TDF. The number of scenarios in which the final funding ratio of $FR^*$ is higher than for the TDF is 257, and the strategies with derisking ($CFR$ and $CFR^*$) end up with a higher funding ratio than the TDF in all scenarios.

At the same time, Panels B-E in Table 3 show that the risk, measured as the volatility and the maximum drawdown of the funding ratio series, is higher for the TDF than the four target income strategies. Indeed, the median funding ratio volatility of the TDF over the first 10 years of the accumulation period, across the 277 scenarios, is 44.6%, while the same value for the four target income strategies is 12.8%, 10.2%, 35.5% and 31% (see Panel B). This result is not surprising, given that the TDF does not include the retirement bond, which is a save asset with zero variations in funding ratio. On the other hand, the target income strategies use the retirement bond to control their level of risk over time. Indeed, the contrasts in the level of risk between the TDF and the target income strategies with derisking, becomes even sharper as the retirement date approaches. The median funding ratio volatility of the TDF is 26% and 44% higher than the one of $CFR$ and $CFR^*$ during the first 10 years of accumulation, while over the last 10 years of accumulation, it is more than twice the level of $CFR$, and 14 times more.

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30The funding ratio volatility is the annualized standard deviation of the monthly series of relative changes of the funding ratio.
volatile than $CFR^*$ over that period\textsuperscript{31}.

Comparing the funding ratios among the target income strategies, we observe two interesting patterns. First, comparing the strategies that include the ratchet mechanism on cumulative pension surplus with the strategies that do not include it (i.e., $FR^*$ vs. $FR$, and $CFR^*$ vs. $CFR$), we observe higher funding ratio quantiles for the low, middle and up to the quantile 75%, while it presents lower values in the right tail of the distributions (quantile 90% and maximum value), as observed in Panel A of Table 3. Second, comparing the strategies with derisking and without derisking (i.e., $FR$ vs. $CFR$, and $FR^*$ vs. $CFR^*$), in Panel A we observe that the strategies with derisking dominate in terms of final funding ratio across all quantiles of the distribution. In terms of risk, they present higher volatility and maximum drawdown during the first 10 years of the accumulation period, across all distribution quantiles (see Panels B and D), but lower risk across all quantiles of these two metrics when measured over the last 10 years of the accumulation phase (see Panels C and E).

The latter patterns are related to the dynamics of the average allocation to equities across scenarios, presented in Figure 5. As observed in the figure, both strategies without derisking ($FR, FR^*$) start with an allocation to the equity index of 30% (i.e., $1 - \kappa$) and remain somewhat close to that level across the accumulation period (albeit with a slight decrease for $FR^*$), while both strategies with derisking start with an allocation to equities of 100% at the start of the accumulation period and decrease at a high rate arriving at a value of 26% at the retirement date for $CFR$, and around 6% for $CFR^*$. Interestingly, this type of pattern in the equity allocation over the lifecycle, starting at 100% and decaying exponentially, is similar to the one proposed by Levy and Levy (2021) (see the bottom panel in Figure 4 in that paper). The main difference between the target income strategies with derisking, and the strategy proposed in Levy and Levy (2021) is that they focus on absolute wealth, instead of retirement income, and as a result, the safe asset in the strategy are short-duration bonds.

5 Discussion of Relation with Optimal Allocation Rules

The target income heuristics proposed in Section 3.2, have common features with several asset allocation strategies in the literature of optimal life-cycle portfolio selection. The first feature is the horizon effect (consisting of a higher allocation to equities for younger investors) in the

\textsuperscript{31}To see this compare the median value of the TDF with the respective values for strategies $CFR$ and $CFR^*$ in panels B and C of Table 3.
asset allocation, which was first referred as a puzzle in Samuelson (1963) but later reconciled with rational portfolio theory in works such as Samuelson (1989), Samuelson (1994), Kim and Omberg (1996), Campbell and Viceira (1999), Barberis (2000), Wachter (2002) and Munk et al. (2004). This feature is clearly reflected in the dynamic allocation rule of target date funds. The horizon-dependent target-income strategies proposed here also incorporate this feature, but unlike the deterministic rule of TDFs, the asset allocation is also state dependent.

A second feature of asset allocation advice documented by Canner et al. (1997), is that professional investment advisors systematically recommend an increasingly higher bonds to stocks ratio for increasingly higher risk aversion. Brennan and Xia (2000) and Munk et al. (2004) find that taking into account interest rate uncertainty introduces this asset allocation feature in the portfolio that maximizes the utility of a rational investor with constant relative risk aversion. The target-income strategies presented in this paper also integrate this feature. Everything else equal, the target-income strategies defined with a higher protection level $\kappa$ parameter have a higher bond-to-equity allocation ratio.

A third feature of optimal asset allocation strategies regards the importance of hedging against changes in the opportunity set, such as variations in interest rates. The importance of hedging portfolios to protect investors from changes in key variables, such as variations in interest rates, is stressed by the literature of portfolio optimization, particularly when the investment horizon is relatively long. Since the classic works of Merton (1969, 1971, 1973), we know that optimal investment strategies include a “speculative term” (i.e. the risky block) and a term that prescribes how the investor should hedge changes in state variables that impact her utility. In accordance with that general prescription, for instance Omberg (1999), Sørensen (1999), Brennan and Xia (2000) and Viceira and Campbell (2001) find that optimal portfolios should hedge variations in interest rates by investing in the zero-coupon bond expiring at the investment horizon of the investor (or a portfolio of regular coupon-paying bonds that replicates the zero-coupon bond). The problem solved in those papers concerns an investor with a consumption objective at a single date in the future. When generalized to a problem when the objective is financing consumption at a series of future dates (the payout period), naturally the hedging asset is a bond that pays indexed cashflows on those dates (or a liability hedging portfolio that aims to replicate the value of the hedging asset, as in Detemple and Rindisbacher, 2008; Martellini and Milhau, 2012). This central piece of advice is widely disregarded by pension

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32 This is precisely the motivation behind the concept of retirement bonds and retirement SeLFIES (see Merton
Also note that the risk-management objective in target income strategies of protecting a floor value of assets defined as a fraction of pension liabilities, is also present in the preferences defined in works such as Sharpe and Tint (1990), Detemple and Rindisbacher (2008), and Martellini and Milhau (2012). Moreover, Martellini and Milhau (2012) finds the asset allocation strategy that maximizes retirement income under a constraint on the minimum level of replacement income and find that “in the presence of funding ratio constraints, the optimal policy is shown to involve dynamic allocation strategies that are reminiscent of portfolio insurance strategies, extended to an asset-liability management (ALM) context”. In general, dynamic allocation based portfolio insurance strategies aim to ensure that the value of the portfolio, denoted here as $A$, stays above a Floor value $F$ at all times, by assigning a proportion of wealth to a risky performance-seeking asset $S$ at every time $t$ equal to,

$$\omega_S(t) = m_t \times \frac{(A(t) - F(t))}{A(t)}, \quad m_t > 0,$$  \hspace{1cm} (21)

while the remaining wealth is invested in the safe or benchmark asset $B$. Using the allocation rule (21), fund managers could insure any of the the floor values of retirement income $F_t = \kappa_t N_t$ implied by the four alternative definitions for $\kappa_t$ introduced in Section 3.2. In other words, the target income rule in Eqs. (10)-(11) is a special case of Eq. (21) with $m_t = 1$ for all $t$. However, using a multiplier $m_t > 1$ introduces “gap risk” to the strategy, which regards the possibility of trespassing the floor value. Indeed, in practice trading can only happen in discrete time and asset prices present “jumps.” Hence, to implement a strategy with $m_t > 1$, it is necessary to determine the maximum value of $m_t$ that would allow $A(t) \geq F(t)$ if an extreme relative loss (of asset value with respect to the floor) happens before the portfolio manager can reallocate assets. For instance, Mantilla-Garcia (2014) develops a methodology to estimate the upper bound of the multiplier in the general case in which the reserve asset $B$ is stochastic, however this introduces model specification and parameter estimation risks to the strategy, which can be avoided by setting $m_t = 1$ at all times.

Another disadvantage of using $m_t > 1$, is that they require trading more frequently, and hence need a high level of liquidity of the underlying assets. Furthermore, they tend to be trend-following, meaning that they sell shares of the risky PSP after the latter experiences a
loss in value relative to the floor process. In presence of mean reverting returns, such strategy would suffer relative to contrarian strategies, or the trend-neutral buy-and-hold (see Perold and Sharpe, 1988). On the other hand, for $m = 1$, the CPPI strategy (21) is actually a buy-and-hold strategy, which means that the strategy is not trend-following nor contrarian, and also that it requires a much lower level of asset liquidity for its implementation. As a consequence, the target income strategies proposed, which imply $m = 1$, do not have the typical trend-following behavior of standard portfolio insurance strategies.\footnote{Indeed, note that for strategies $FR$ and $CFR$ trading only happens whenever there are new contributions to the fund. For strategies $FR^*$ and $CFR^*$, their ratchet effect in fact can induce a contrarian trading behavior during periods of strong outperformance of the PSP.}

On the other hand, the disadvantage of using $m_t = 1$ at all times is that the exposure to the upside potential of risky assets can be relatively low, especially if the floor definition is similar to the standard CPPI floor i.e. $F_t = \kappa N_t$, which is the case of strategy $FR$.\footnote{The standard CPPI floor is actually $F_t = \kappa B_t$, which is the same as $F_t = \kappa N_t$ for a unique initial contribution to the fund. In other words, the CPPI floor formula was generalized for multiple contributions.} However, by discounting the expected contributions related to future labor income in the floor definition, the strategies with horizon-derisking ($CFR$ and $CFR^*$) are able to have a large exposure to risky assets during the initial years of the accumulation phase, while still avoiding the gap risk.\footnote{To see how strategy $CFR$ discounts expected contributions $\tilde{n}_i$, note that the lower bound on $\tilde{N}_t$ implied by $\kappa_t^{CFR}$ can be written as $\tilde{N}_t \geq \kappa \tilde{N}_T - \sum_{t'+1}^{T} \tilde{n}_{t'}$.}

Interestingly, in our empirical simulation of the strategies, the average allocation to equities across scenarios exhibits a very similar pattern over the accumulation phase, to the allocation of the optimal strategy in Levy and Levy (2021), which also integrate expected contributions coming from future labor income (compare the bottom panel in Figure 4 in that paper with Figure 5).

Given an investable alternative for the retirement bonds (or a replicating portfolio), the target-income strategies proposed are heuristics that are very simple to implement. In that sense, they constitute an alternative to the current glide-path strategies followed by target date funds. However, the target-income strategies not only integrate the horizon effect, but also other key lessons from the optimal portfolio theory aforementioned. Furthermore, relative to optimal allocation rules, the target income strategies have the advantage of not relying in any parameter estimates or model assumptions on asset’s dynamics in order to insure an increasing
minimum level of retirement income.

6 Conclusion

The price of retirement bonds (or its replicating portfolio) is an instrumental piece of information since it enables individual investors to know: i. how much replacement income the current value of their retirement assets can secure at any point during the accumulation period, and ii. how much retirement income they could have afforded risk-free, have all their previous contributions been invested in retirement bonds (denoted \( N_t \)). We leverage the latter piece of information to provide a formal measure of pension liabilities for DC plans, which we define as the present value of the retirement income benefits that each individual could have secured with their previous contributions, without taking any risk.

Using the proposed DC liability definition and the corresponding funding ratio as performance metric, creates a crucial incentive for pension fund managers to control the primary investment risk their affiliates face: the variations in the exchange rate of assets to retirement income. Indeed, using historical data from U.S. government bond yields to value retirement bonds, we show that, when assets’ value is measured in units of retirement benefits (which is what matters for pension fund affiliates), an equity index and a bond portfolio with relatively short and constant duration, have the same level of risk on average. This fact has important implications for the design of life-cycle products for the accumulation phase. Taking into account the variations in the exchange rate of assets to retirement income, we find that the popular target date funds, which use short-duration bonds as the “safe” asset in their asset allocation strategy, do a poor job at reducing the variations in the expected level of retirement income benefits, as the retirement date approaches.

As an alternative, we introduce a series of target income strategies, which on the one hand secure an essential level of retirement income regardless of the fluctuations in the risky assets and interest rates, but at the same time invest in risky securities to pursue the aspirational goal of a higher income, beyond the level affordable risk-free. Given an investable alternative for the retirement bond (such as a replicant portfolio), it is straightforward for pension fund management companies to secure a proportion \( \kappa_t \) of the affordable retirement income \( N_t \), while accessing part of the upside potential of risky assets, by simply holding a number of retirement bonds equal to \( \kappa_t N_t \), and investing the remaining capital in any risky performance-seeking port-
folio of choice. We propose four different functions for the (possibly) time-varying proportion $\kappa_t$ to secure, in order to integrate common principles in investment advice, such as decreasing risk as the retirement date approaches (in terms of retirement units), and/or securing a proportion of cumulative gains from risky investments over time (i.e., a ratchet effect on retirement benefits).

Using our definition of pension liability for DC plans, we introduce a flexible and simple payment function for pension fund managers, that align their incentives with the aspirational and essential retirement income goals of the fund affiliates. This fee structure directly connects with the target income strategies, and its use would constitute a strong incentive to foster the dire retirement income security needed for DC plan beneficiaries.

References


**Appendix A  Pricing of retirement bonds**

The no-arbitrage price of the retirement bond at a date $t$ in the accumulation phase is:

$$B_t = \sum_{h=1}^{M} (1 + \pi)^{T+h} e^{-(T+h-t)R_{t,T+h-t}}$$  \hspace{1cm} (22)

where cashflows are paid on $T+h$ periods ahead for $h$ in \{1, 2, .., $M$\}, where $T$ is the initial time to retirement, $M$ the expected duration of the payout phase (e.g., 20 years), $R_{t,T+h-t}$ denotes the yield of a zero bond paying $1$, $T + h - t$ years ahead, and $\pi$ is a COLA rate (e.g., 2%
annually). The discount rates can be inferred from coupon-paying bond prices through standard bootstrapping techniques or fitting them to yield curve models such as Nelson and Siegel (1987). Note that the fixed COLA adjustment only covers the impact of expected inflation. If an exact hedge of inflation risk is preferred, then the cashflows would be expressed in real terms, which simply means that the discount rates $R_{T,h}$ in equation (22) should be real yields instead of nominal bond yields.

If a hedge for salary increments in real terms was preferred as well, then real yields would be used as discount rates in Eq. (22), and $\pi$ would be the expected salary growth in real terms. However, having extra hedging for unexpected inflation risk and matching an expected real salary growth imply an increment in the cost of financing future consumption, reflected in a higher price $B_t$. We leave the analysis of the cashflow indexing tradeoff in retirement bonds for future research. In our empirical section, we opt for using nominal yields (and set $\pi$ as the inflation target of the Federal Reserve), in order to have the longest available time period for the analysis.

Merton and Muralidhar (2017) argue that sovereign states should issue another type of closed-related bonds, which they call retirement SeLFIES for Standard of Living indexed, Forward-starting, Income-only Securities, and make them available to investors in accumulation (see also Muralidhar, 2015; Muralidhar et al., 2016; Kobor and Muralidhar, 2018; Martellini et al., 2018). SeLFIES are similar to the retirement bonds above, except that their cash-flows are indexed to aggregate per capita consumption. One key advantage of this feature is that it provides investors with a hedge of standard-of-living risk, while inflation-linked retirement bonds only provide a hedge against cost-of-living risk. This feature, however, introduces a form of market incompleteness and makes SeLFIES non-redundant securities that existing fixed-income instruments cannot replicate.

**Appendix B  Dynamics of fund value in retirement units $\tilde{N}_t$**

Denote the returns of the pension fund assets $r^A$, thus we have that the value of retirement assets at each point in time is:

$$A_{t+1} = A_t(1 + r^A_{t+1}) + C_{t+1}$$  \hspace{1cm} (23)
where $C_{t+1}$ is the contribution to the fund at time $t+1$. The affordable replacement income evolves by definition as

$$\Delta N_{t+1} = N_{t+1} - N_t = n_{t+1} = \frac{C_{t+1}}{B_{t+1}}$$

(24)

Hence $N_{t+1}$ never decreases, and only increases with new contributions measured in retirement units, i.e., $n_{t+1}$. On the other hand, the value change of a pension fund invested in any kind of assets, measured in terms of retirement units is:

$$\tilde{n}_{t+1} := \Delta \tilde{N}_{t+1} = \tilde{N}_{t+1} - \tilde{N}_t = \frac{A_{t+1}}{B_{t+1}} - \frac{A_t}{B_t}$$

(25)

Replacing (23) in (25) we have,

$$\tilde{n}_{t+1} = \frac{A_t(1 + r_{t+1}^A)}{B_{t+1}} + \frac{C_{t+1}}{B_{t+1}} - \frac{A_t}{B_t}$$

(26)

$$\tilde{n}_{t+1} = \tilde{N}_t \left( \frac{1 + r_{t+1}^A}{1 + r_{t+1}^B} - 1 \right) + n_{t+1}$$

(27)

The latter can be expressed in the following way by replacing $B_{t+1} = B_t(1 + r_{t+1}^B)$ in the denominator and recalling that $\tilde{N}_t = \frac{A_t}{B_t}$ and $n_{t+1} = \frac{C_{t+1}}{B_{t+1}}$:

$$\tilde{n}_{t+1} = \tilde{N}_t \left( 1 + r_{t+1}^A \right) - \frac{A_t}{B_t}$$

(28)

Notice that $\tilde{n}_{t+1} - n_{t+1}$ is proportional to the percentage change in the relative value of the assets with respect to the price of the retirement bond. To see this, define $A_{t+1}' := A_t'(1 + r_{t+1}^A)$ with $A_0' = A_0$, i.e., the cumulative returns of investments in the pension fund, and $Z_t := \frac{A_t'}{B_t}$, $\forall t$ and notice that $r_{t+1}^Z = \frac{Z_{t+1}}{Z_t} - 1 = \frac{1 + r_{t+1}^A}{1 + r_{t+1}^B} - 1$, which are the relative returns of the investments in the pension fund. Thus, using definition (25) in equation (27) we obtain:

$$\tilde{N}_{t+1} = \tilde{N}_t(1 + r_{t+1}^Z) + n_{t+1}$$

and

$$\tilde{n}_{t+1} = \tilde{N}_t r_{t+1}^Z + n_{t+1}$$

(28)
Appendix C  Condition on the dynamics of $\kappa_t$

Following the allocation rule (10)-(11), the dynamics of $\tilde{N}_t$ are

\[ \tilde{N}_{t+1} \geq \kappa_t N_t + (\tilde{N}_t - \kappa_t N_t)(1 + r_{PSP}^{Z_{t+1}}) + n_{t+1} \]  

where $r_{PSP}^{Z_{t+1}} = \frac{1 + r_{PSP}^{PSP}}{1 + r_{B}^{PSP}} - 1$ is the relative return of the PSP with respect to the return of the retirement bond. Replacing Eq. (29) in the liability-driven constraint (4),

\[ \tilde{N}_{t+1} \geq \kappa_{t+1} N_{t+1} \]

\[ \kappa_t N_t + (\tilde{N}_t - \kappa_t N_t)(1 + r_{PSP}^{Z_{t+1}}) + n_{t+1} \geq (\kappa_t + \Delta \kappa_t) N_t + \kappa_{t+1} n_{t+1} \]

\[ (\tilde{N}_t - \kappa_t N_t)(1 + r_{PSP}^{Z_{t+1}}) \geq \Delta \kappa_t N_t - (1 - \kappa_{t+1}) n_{t+1} \]  

Note that since $r_{PSP}^{PSP} > -1$, then $r_{PSP}^{Z_{t+1}} > -1$. Hence, if at the previous time period the floor income was respected, i.e., $\tilde{N}_t - \kappa_t N_t \geq 0$, then the left hand side of Eq. (31) is positive, which means that a sufficient condition to meet condition (30) on $t + 1$ would be:

\[ \Delta \kappa_t N_t - (1 - \kappa_{t+1}) n_{t+1} \leq 0 \]

\[ \Delta \kappa_t N_t - n_{t+1} + (\kappa_t + \Delta \kappa_t) n_{t+1} \leq 0 \]

\[ \Delta \kappa_t N_{t+1} - (1 - \kappa_t) n_{t+1} \leq 0 \]

\[ \Delta \kappa_t \leq (1 - \kappa_t) \frac{n_{t+1}}{N_{t+1}} \]  

if at the previous period $\tilde{N}_t - \kappa_t N_t \geq 0$ held. Recall that by convention $\tilde{N}_0 = N_0$, hence the condition holds at the initial time $t = 0$, i.e., $\tilde{N}_0 - \kappa_0 N_0 \geq 0$ for $\kappa_0 \leq 1$. Hence, iteratively, the floor condition Eq. (30) is met for every $t \geq 1$ if the increase in $\kappa$ satisfies Eq. (32), which is Eq. (12) in the main text.

Appendix D  Condition check on $\kappa_t^{CFR}$

Rewrite condition (12) as

\[ \Delta \kappa_t N_{t+1} \leq (1 - \kappa_t) n_{t+1} \]  

40
On the one hand, given definition (12) of $\kappa_{t}^{CFR}$, we have:

$$\Delta \kappa_{t}^{CFR} = (1 - \kappa) \tilde{N}_{T} \left( \frac{1}{N_{t}} - \frac{1}{N_{t+1}} \right)$$

$$\Delta \kappa_{t}^{CFR} N_{t+1} = (1 - \kappa) \tilde{N}_{T} \left( \frac{N_{t+1} - N_{t}}{N_{t}} \right)$$

$$\Delta \kappa_{t}^{CFR} N_{t+1} = (1 - \kappa) \tilde{N}_{T} \left( \frac{n_{t+1}}{N_{t}} \right) \tag{34}$$

On the other hand, replacing the definition (12) of $\kappa_{t}^{CFR}$ on the right hand side of condition (33), we obtain the same expression as Eq. (34). Hence, $\kappa_{t}^{CFR}$ satisfies the condition (12) with equality.

Appendix E  Condition check on $\kappa_{t}^{FR*}$

First consider the case in which a new maximum surplus $S_{t}^{*} = \tilde{N}_{t}^{*} - N_{t}^{*}$ is attained, hence $t = t^{*}$. The condition to be satisfied is

$$\tilde{N}_{t}^{*} \geq \kappa_{t}^{FR*} N_{t}^{*}$$

$$\tilde{N}_{t}^{*} \geq \kappa \left( 1 + \frac{S_{t}^{*}}{N_{t}^{*}} \right) N_{t}^{*}$$

$$\tilde{N}_{t}^{*} \geq \kappa (N_{t}^{*} + S_{t}^{*})$$

$$\tilde{N}_{t}^{*} \geq \kappa \tilde{N}_{t}^{*}$$

which is satisfied for $\kappa \leq 1$. Now consider the other case in which the surplus’ running maximum is still the same as in the previous period, i.e., $S_{t+1}^{*} = S_{t}^{*}$. On the left hand side of the equivalent condition (33), given definition (16) of $\kappa_{t}^{FR*}$, we have:

$$\Delta \kappa_{t}^{FR*} = \kappa \left( \frac{S_{t+1}^{*}}{N_{t+1}^{*}} - \frac{S_{t}^{*}}{N_{t}^{*}} \right)$$

$$\Delta \kappa_{t}^{FR*} = \kappa S_{t}^{*} \left( \frac{1}{N_{t+1}} - \frac{1}{N_{t}} \right)$$

$$\Delta \kappa_{t}^{FR*} N_{t+1} = \kappa S_{t}^{*} \left( \frac{N_{t} - N_{t+1}}{N_{t}} \right)$$

$$\Delta \kappa_{t}^{FR*} N_{t+1} = -\kappa S_{t}^{*} \left( \frac{n_{t+1}}{N_{t}} \right) \tag{35}$$
On the other hand, replacing the definition of $\kappa^{FR^*}_t$ on the right hand side of condition (33), we have

$$
(1 - \kappa^{FR^*}_t) n_{t+1} = \left( 1 - \kappa \left( 1 + \frac{S^*_t}{N^*_t} \right) \right) n_{t+1}
$$

(36)

Note that the last term in Eq. (36) is the same as the rhs of Eq. (35). Hence, for $0 \leq \kappa \leq 1$ $\kappa^{FR^*}_t$ satisfies the condition (12).

**Appendix F  Condition check on $\kappa^{CFR^*}_t$**

First consider the case in which a new maximum surplus $S^*_t = \tilde{N}_t - N_t^*$ is attained, hence $t = t^*$. The condition to be satisfied is

$$
\tilde{N}_t^* \geq \kappa^{CFR^*}_t N_t^*,
$$

$$
\tilde{N}_t^* \geq \left( \kappa_t^* + \kappa^* \frac{S^*_t}{N^*_t} \right) N_t^*,
$$

$$
\tilde{N}_t^* \geq \kappa_t^* \tilde{N}_t^* + \kappa S^*_t,
$$

$$
\tilde{N}_t^* \geq \left( 1 - (1 - \kappa) \frac{N_t^* + \kappa S^*_t}{N_t^*} \right) N_t^* + \kappa S_t^*,
$$

$$
\tilde{N}_t^* \geq N_t^* - (1 - \kappa) \tilde{N}_T + \kappa S_t^*,
$$

$$
\tilde{N}_t^* \geq \kappa \tilde{N}_t^* + (1 - \kappa)(N_t^* - \tilde{N}_T)
$$

$$
\tilde{N}_t^*, (1 - \kappa) \geq (1 - \kappa)(N_t^* - \tilde{N}_T)
$$

$$
\tilde{N}_t^*, - N_t^* \geq - \tilde{N}_T
$$

(37)

where above we assumed $0 \leq \kappa \leq 1$. Since $\tilde{N}_0 = N_0$, then for any $t > 0$, $\tilde{N}_t \geq N_t$, so condition (37) is satisfied.

Now consider the other case in which the surplus’ running maximum is still the same as in the previous period, i.e., $S^*_{t+1} = S^*_t$. On the left hand side of the equivalent condition (33), given definition (17) of $\kappa^{CFR^*}_t$, we have:

$$
\Delta \kappa^{CFR^*}_t N_{t+1} = \left( \Delta \kappa^{CFR}_t + \Delta \kappa^{FR^*}_t \right) N_{t+1}
$$

(38)
Replacing Eq. (34) and Eq. (35) on Eq. (38),

\[
\Delta \kappa_{CFR}^* N_{t+1} = (1 - \kappa) \hat{N}_T \left( \frac{n_{t+1}}{N_t} \right) - \kappa S_t^* \left( \frac{n_{t+1}}{N_t} \right)
\]

(39)

On the other hand, replacing the definition of \( \kappa_{CFR}^* \) on the right hand side of condition (33), we have

\[
(1 - \kappa_{CFR}^*) n_{t+1} = \left( (1 - \kappa) \frac{\hat{N}_T}{N_t} - \kappa \left( \frac{S_t^*}{N_t} \right) \right) n_{t+1}
\]

(40)

Note that the Eq. (40) is equal to Eq. (39). Hence, for \( 0 \leq \kappa \leq 1 \), \( \kappa_{CFR}^* \) satisfies the condition (12) with equality.

**Appendix G  Bound on funding ratio drawdown of \( FR^* \) and \( CFR^* \)**

The target income strategy \( CFR^* \) ensures at all times that,

\[
\tilde{N}_t \geq \kappa_{CFR}^* N_t
\]

Hence, the funding ratio of the strategy, \( FR_t \), is bounded from below:

\[
FR_t \geq \kappa_{CFR}^*
\]

\[
FR_t \geq \kappa_{CFR}^* + \kappa \left( \frac{S_{t^*}}{N_t} \right)
\]

\[
FR_t \geq \kappa_{CFR}^* + \kappa \left( \frac{\tilde{N}_{t^*}}{N_t} - \frac{N_{t^*}}{N_t} \right)
\]

43
if there are no contributions from $t^*$ until $t$, then $N_t = N_t^*$, and

$$FR_t \geq \kappa_{t}^{CFR} + \kappa \left( \frac{\hat{N}_t^*}{N_t^*} - 1 \right)$$

$$FR_t \geq \kappa_{t}^{CFR} + \kappa FR_t^* - \kappa$$

(41)

$$FR_t \geq 1 - (1 - \kappa) \frac{\hat{N}_t^*}{N_t^*} + \kappa FR_t^* - \kappa$$

$$FR_t \geq (1 - \kappa) \left( 1 - \frac{\hat{N}_t^*}{N_t^*} \right) + \kappa FR_t^*$$

$$\frac{FR_t}{FR_t^*} - 1 \geq (\kappa - 1) \left( \frac{\hat{N}_t^*}{N_t^*} - 1 \right) \frac{1}{FR_t^*} + \kappa - 1$$

$$\frac{FR_t}{FR_t^*} - 1 \geq (\kappa - 1) \left( \frac{\hat{N}_t^* + \hat{N}_T - N_t^*}{N_t^*} \right)$$

(42)

Hence, whenever there are no contributions from $t^*$ on, but they had reached or passed the target level by that time, i.e., $N_t^* \geq \hat{N}_T$, then the ratio to the right of Eq. (42) is lower than 1, which means that the max drawdown\(^{36}\) in funding ratio of strategy $CFR^*$ is bounded by $\kappa - 1$.

From the definition, of $\kappa_{t}^{FR}$, it follows that the lower bound of the funding ratio for the strategy $FR^*$ is similar to Eq. (41) but with $\kappa$ instead of $\kappa_{t}^{CFR}$, whenever there are no contributions from $t^*$ on. Hence, we have:

$$FR_t \geq \kappa + \kappa FR_t^* - \kappa$$

$$FR_t \geq \kappa FR_t^*$$

$$\frac{FR_t}{FR_t^*} - 1 \geq \kappa - 1$$

(43)

Hence, whenever there are no contributions from $t^*$ on, the max drawdown in funding ratio of strategy $FR^*$ is bounded by $\kappa - 1$.

### Appendix H Tables and Figures

\(^{36}\)The max drawdown is the maximum percentage drop observed in the funding ratio, until current time $t$, relative to its running maximum $FR_t^*$ (i.e., lhs of Eq. 42).
Figure 1: Yield Curve Parameters for Nelson Siegel Model. We use the ‘data-based’ definitions in Diebold and Li (2006), setting $\lambda = 0.0609 \times 12$, the level factor is proxied by the 10-year yield, the slope as the difference between the 10-year and the short-term yield, and the curvature as the twice the medium-term yield minus the sum of the short-term and 10-year yields. For periods for which the 2-year yield (‘TCMNOM_Y2’) is not available (1962-01-02 to 1976-05-31), we use the 5-year yield (‘TCMNOM_Y5’) as the medium term rate instead, and for periods for which the CRSP 3-month yield (‘NFCP_M3’) is not available (1962-01-02 to 1997-01-01) we use the 1-year yield (‘TCMNOM_Y1’) as the short-term rate.
Table 1: Distribution summary across all accumulation periods of assets’ annualized return and volatility, correlation with the corresponding retirement bond (R.bond), and relative drawdown (RDD) with respect to the R.bond. All figures are in percentage terms.

<table>
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<tr>
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<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
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Figure 2: Cumulative Returns for the equity index, the 5-year constant duration bond index, and the retirement bond corresponding to the cohort retiring at the end of each sample period. The left Panel presents the data corresponding to the cohort retiring in 1996-12-31, and the right Panel for the cohort retiring in 2019-12-31.

Table 2: Distribution summary across all simulated cohorts of the secured proportion of affordable retirement income, $\kappa_T$, obtained from investing 12.4% of the average labor income, for each of the age-income profiles.

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<th>50%</th>
<th>75%</th>
<th>90%</th>
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Figure 3: Securable monthly retirement income dollars from investing the monthly contributions in the target date fund or the target-income strategies with $\kappa = 0.7$. The figure also presents the lower bound protected by the target-income strategies. The left panels correspond to the historical simulation from January 1962 to December 1996. The right panels correspond to the historical simulation from January 1990 to December 2019.
Table 3: Distribution summary across all accumulation periods for the target date fund and target income strategies’ final funding ratio (FR) across the 277 cohorts, FR volatility over the first 10 years of accumulation, FR volatility over the last 10 years of accumulation, and maximum drawdown (MDD) of FR over the first 10 years of accumulation and over the last 10 years of accumulation. All figures are in percentage terms.

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Figure 4: Funding ratio of target date fund (TDF) and of funding ratio of the target-income strategies with $\kappa = 0.7$. The figure also presents the lower bound protected by the target-income strategies. The left panels correspond to the historical simulation from January 1962 to December 1996. The right panels correspond to the historical simulation from January 1990 to December 2019.
Figure 5: Average equity allocation across all 277 scenarios of the target date fund (TDF) and the target income strategies.